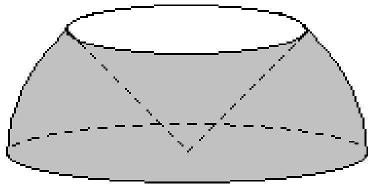
Terceira Prova de CDI-II – 26/mai/2007 – 09:30



1) Se $\boldsymbol{d}(x,y,z)$ representa a densidade (g/cm^3) de um corpo heterogêneo, então a integral de $\boldsymbol{d}(x,y,z)$ numa certa região do $\boldsymbol{R^3}$ resulta na massa desta região.
Monte (em **dois sistemas** diferentes de coordenadas) a integral que calcula a massa da região dada pela figura ao lado (meia esfera de raio $10\ cm$, de onde foi extraído um cone com ângulo de 90^o)

$$\boldsymbol{d}(x,y,z) = \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$$

2) Transformar em coordenadas retangulares $\displaystyle\int_0^{\frac{\pi}{2}}\int_{2cos\theta}^{2}(4\rho-\rho^3)d\rho d\theta$

3) Monte a integral que calcula o volume do sólido limitado pelas superfícies $4z+4x^2+y^2 = 16$ e $12x^2+y^2 = 4z$

4) Escrever em coordenadas cilíndricas ou esféricas

$$\int_0^3\int_0^{\sqrt{4-x^2}}\int_{2x^2+2y^2}^{\sqrt{8-x^2-y^2}}\sqrt{9+x^2+y^2}\,dzdydx$$

5) Calcule uma das integrais referentes a uma das questões anteriores.

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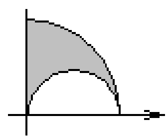
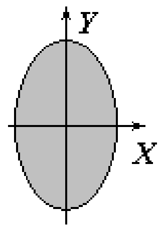
Gabarito:

1) $m = \displaystyle\int_0^{2\pi}\int_0^{10}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} r^2sen^2\phi\, d\phi\, dr\, d\theta$

$$m = \int_0^{2\pi}\int_0^{5\sqrt{2}}\int_0^{\rho}\frac{\rho^2}{\sqrt{\rho^2+z^2}}\,dz\,d\rho\,d\theta + \int_0^{2\pi}\int_{5\sqrt{2}}^{10}\int_0^{\sqrt{\rho^2+z^2}}\frac{\rho^2}{\sqrt{\rho^2+z^2}}\,dz\,d\rho\,d\theta$$

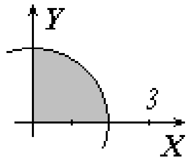


2) $\displaystyle\int_0^2\int_{\sqrt{2x-x^2}}^{\sqrt{4-x^2}}(4 - x^2 - y^2)\,dy\,dx$



3) $4z= 16 - 4x^2 - y^2 = 12x^2+y^2 \Rightarrow 16x^2+ 2y^2 = 16 \Rightarrow y^2 = 8 - 8x^2$

$$V = \int_0^1\int_0^{\sqrt{8-8x^2}}(16x^2+2y^2-16)\,dydx$$



4) $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^2\int_{2\rho^2}^{\sqrt{8-\rho^2}}\sqrt{9+\rho^2}\ \rho\, d\rho\, d\theta$

5) $m = \displaystyle\int_0^{2\pi}\int_0^{10}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} r^2sen^2\phi\, d\phi\, dr\, d\theta = \frac{1}{2}\int_0^{2\pi}\int_0^{10} r^2\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}(1\text{-}cos2\phi)\, d\phi\, dr\, d\theta =$

$$\frac{1}{2}\int_0^{2\pi}\int_0^{10} r^2\left[\phi - \frac{sen\,2\phi}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} dr\,d\theta = \frac{\pi+2}{8}\int_0^{2\pi}\int_0^{10} r^2\,dr\,d\theta == \frac{\pi+2}{8}\int_0^{2\pi}\left[\frac{r^3}{3}\right]_0^{10} d\theta =$$

$$\frac{\pi+2}{8}\frac{1000}{3}2\pi = \frac{250\pi(\pi+2)}{3}$$