Quarta Prova de CDI-II - 2 de julho de 2007 - noite

1) Verifique (, com justificativas) o tipo de convergência (ou não) de:

$a)\ \sum \dfrac{\cos n\pi}{n}$ e $b)\ \sum \left(\dfrac{2n+1}{3n-2}\right)^n$

2) Encontre o valor de:

$a)\ \displaystyle\sum_{n=1}^{50} \frac{4}{\sqrt{n+2}+\sqrt{n}}$ e $b)\ \displaystyle\sum_{n=1}^{30} \frac{2^{n+1}}{5.3^n}$

3) Verifique o domínio de convergência de:

$$\sum \frac{(5x)^n}{2^n(4+n)}$$

4) Desenvolver em séries de *potências*: ***sen²3x***

5) Calcule com *erro* $< 10^{-3}$: $\displaystyle\int_0^1 e^{-x^3}\,dx$

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Gabarito:

1) a) $\dfrac{\cos n\pi}{n} = \dfrac{(-1)^n}{n} \rightarrow \sum \dfrac{\cos n\pi}{n}$ *é condicionalmente convergente.*

b) $\displaystyle\lim_{n\to\infty} \sqrt[n]{\left(\frac{2n+1}{3n-2}\right)^n} = \frac{2}{3} < 1 \rightarrow \sum \left(\frac{2n+1}{3n-2}\right)^n$ *converge.*

2) a) Série Telescópica: $\dfrac{4}{\sqrt{n+2}+\sqrt{n}} = 2\,(\sqrt{n+2} - \sqrt{n}\,) \rightarrow \displaystyle\sum_{n=1}^{50} \frac{4}{\sqrt{n+2}+\sqrt{n}} =$

$2[(\sqrt{3} - \sqrt{1}\,)+(\sqrt{4} - \sqrt{2}\,)+(\sqrt{5} - \sqrt{3}\,)+\dots\ (\sqrt{51} - \sqrt{49}\,)+(\sqrt{52} - \sqrt{50}\,)] =$

$2[-\sqrt{1} - \sqrt{2} + \sqrt{51} + \sqrt{52}\,] = 2[\sqrt{52} + \sqrt{51} - \sqrt{2} - 1] \sim 23{,}88.$

b) Série Geométrica: $S_{30} = \dfrac{2}{5}\,\dfrac{\left(\frac{2}{3}\right)^{31} - \frac{2}{3}}{\frac{2}{3} - 1} = \dfrac{2}{5}\,\dfrac{\frac{2}{3}\left[\left(\frac{2}{3}\right)^{30} - 1\right]}{-\frac{1}{3}} =$

$\dfrac{4}{5}\left[1 - \left(\dfrac{2}{3}\right)^{30}\right] \sim 0{,}799996.$

3) $\displaystyle\lim_{n\to\infty} \left| \frac{(5x)^n(5x)}{2^n 2(5+n)}\,\frac{2^n(4+n)}{(5x)^n} \right| = \lim_{n\to\infty} \left| \frac{(5x)(4+n)}{10+2n} \right| = |5x|\frac{1}{2} < 1 \rightarrow |5x| < 2$

$-2 < 5x < 2 \rightarrow -\dfrac{2}{5} < x < \dfrac{2}{5}$

Para x = - $\frac{2}{5}$, *temos* $\sum \frac{(-1)^n}{(4+n)}$, *alternada convergente por Libnitz e*

Para x = , $\frac{2}{5}$ *temos* $\sum \frac{1}{(4+n)}$, *divergente (harmônica).*

Dom Conv. = $\left[-\frac{2}{5}, \frac{2}{5}\right)$.

4) $f(x) = sen^2 3x \rightarrow f(0) = 0 \rightarrow a_0 = 0$

$f'(x) = 2.3\ sen^2 3x \cos 3x = 3.sen\ 6x \rightarrow f'(0) = 0 \rightarrow a_1 = 0$

$f''(x) = 3.6.\cos 6x \rightarrow f''(0) = 0 \rightarrow a_2 = \frac{3.6}{2!}$

$f'''(x) = -3.6^2.sen\ 6x \rightarrow f'''(0) = 0 \rightarrow a_2 = 0$

$f^{iv}(x) = -3.6^3.\cos 6x \rightarrow f^{iv}(0) = -3.6^3 \rightarrow a_2 = -\frac{3.6^3}{4!}$

$f(x) = sen^2 3x = \frac{3.6}{2!}x^2 - \frac{3.6^3}{4!}x^4 + \ldots = \sum \frac{(-1)^{n-1} 3.6^{2n-1} x^{2n}}{(2n)!}$

5) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \rightarrow e^{-x^3} = \sum_{n=0}^{\infty} \frac{(-x^3)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{3n}}{n!} \rightarrow$

$$\int_0^1 e^{-x^3} dx = \left[\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{n!(3n+1)}\right]_0^1 = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(3n+1)} =$$

$$\frac{1}{1} - \frac{1}{4} + \frac{1}{2.7} - \frac{1}{6.10} + \frac{1}{24.13} - \frac{1}{120.16} \sim 0,807.$$