9ª. LISTA – FUNÇÕES DE VÁRIAS VARIÁVEIS CÁLCUL O II

1. Dada as funções $f(x,y)=\frac{x}{4}-\frac{\sqrt[3]{xy}}{y^2}$ e $g(x,y,t)=(x+y)^2-\frac{t}{xy}+5$ determine:

a) $f(2\,,4)$ b) $f(t^2\,,t)$ c) $g(-1,2,4)$ d) $g(s,s^2,s^3)$

2. Determine e esboce o domínio da região:

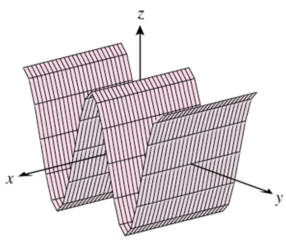
a) $f(x,y)=ln(1-x^2-y^2)$ b) $f(x,y)=\frac{1}{\sqrt{y-x^2}}$ c) $f(x,y)=3e^{\sqrt{y-x}}$

3. Esboce o gráfico de curvas de nível (mapa de contorno) com k={1,2,3,4} para:

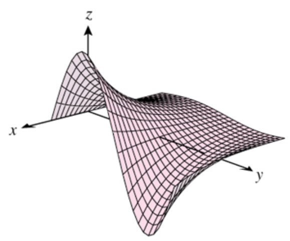
a) $f(x,y)=xy$ b) $f(x,y)=x^2+y$ c) $f(x,y)=\frac{\sqrt{x^2+y^2}}{2}$

4. Dadas as funções, associe as respectivas suas curvas de nível (mapas de contornos)

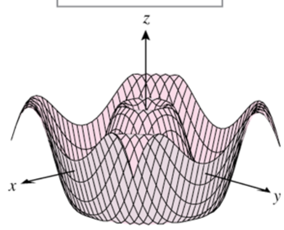
a) $z=\cos y$



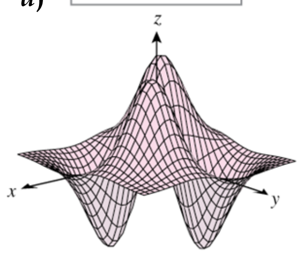
b) $z=e^x \operatorname{sen} y$



c) $z=\operatorname{sen}(\sqrt{x^2+y^2})$



d) $z=xye^{-\frac{1}{2}(x^2+y^2)}$



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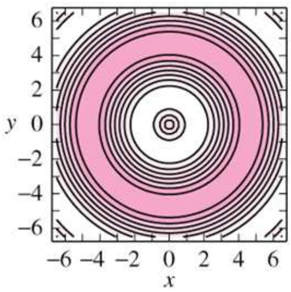

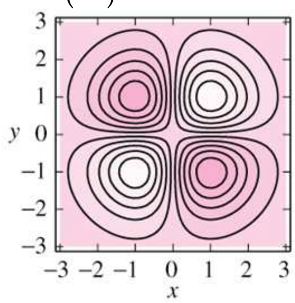

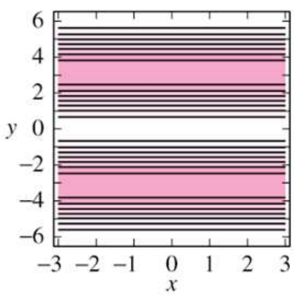

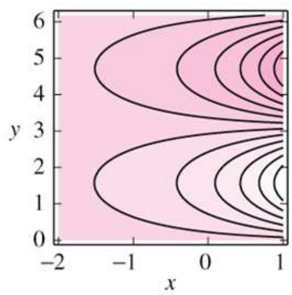

5. Dadas as funções, associe as respectivas suas superfícies de nível

a) $f(x,y,z)=x+z$ b) $f(x,y,z)=x^2+y^2+z^2$ c) $f(x,y,z)=-\,y^2+z$ d) $f(x,y,z)=-x^2-y^2+z$

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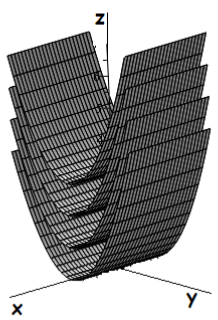
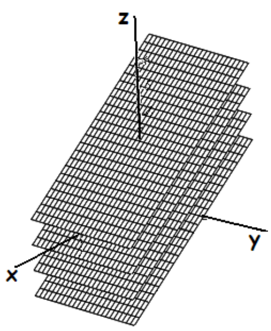
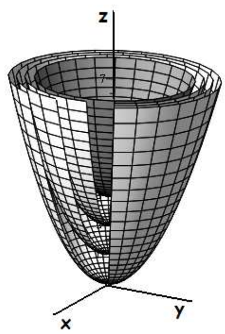
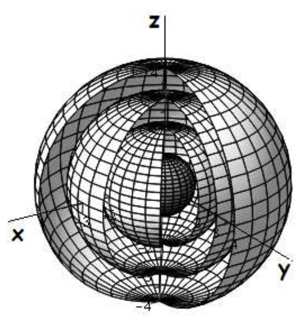








6. Determine as derivadas parciais solicitadas por definição:

a) $z = 5xy - x^2$, $\frac{\partial z}{\partial x}$ b) $f(x,y) = x^2 + y^2 - 10$, $\frac{\partial f}{\partial y}$ c) $z = 2x + 5y - 3$, $\frac{\partial z}{\partial x}$ e $\frac{\partial z}{\partial y}$

7. Dada a função, determine as derivadas parciais solicitadas:

a) $f(x,y) = 3x^3y^2$, $f_x(x,y)$, $f_y(x,y)$, $f_x(1,2)$, $f_y(1,2)$

b) $z = e^{2x} seny$, $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \left.\frac{\partial z}{\partial x}\right|_{(\ln 2,0)}, \left.\frac{\partial z}{\partial y}\right|_{(\ln 2,0)}$

c) Seja $z = \sqrt{x}.\cos y$, $\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y \partial x}$

d) $f(x,y) = e^{x^2 y}$, $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial x \partial y}$

e) $f(x,y) = \cos(x^5 y^4)$, $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x \partial y}$

f) $f(x,y) = e^{xy} sen\, 4y^2$, $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$

g) $f(x,y) = \frac{x\,y}{x^2 + y^2}$, $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$

h) $f(x,y) = \frac{x+y}{x-y}$, $f_x(x,y)$, $f_y(x,y)$

i) $f(x,y) = x^2 y\, e^{xy}$, $\frac{\partial f}{\partial x}(1,1)$, $\frac{\partial f}{\partial y}(1,1)$

j) $w = x^2 cos\, xy$, $\left.\frac{\partial w}{\partial x}\right|_{(\frac{1}{2},\pi)}$, $\left.\frac{\partial w}{\partial y}\right|_{(\frac{1}{2},\pi)}$

k) $h(x,y) = ln(4x - 5y)$, $\frac{\partial^2 h}{\partial x^2}, \frac{\partial^2 h}{\partial y^2}$

l) $f(x,y,z) = x^3y^5z^7 + xy^2 + y^3z$, f_{xy} , f_{xz} f_{xyy} , f_{xyz} , f_{xxyz}

8. Use a regra da cadeia para determinar $\frac{df}{dt}$

a) $f(x,y) = ln(x^2 + y)$, $x = 2t + 1$, $y = 4t^2 - 5$

b) $f(x,y) = sen(2x + y)$, $x = cos\,t$, $y = sen\,t$

c) $f(x,y) = 4x\, e^{xy^2}$, $x = 2t$, $y = \frac{t^2}{4}$

d) $f(x,y) = \ln xy$, $x = 2t^2$, $y = t^2 + 2$

9. Use a regra da cadeia para determinar: $\frac{dz}{du}$ e $\frac{dz}{dv}$

a) $z = \sqrt{x^2 + y^3}$, $x = u^2$, $y = \sqrt[3]{v^2}$

b) $z = -ln(x^2 - y^2)$, $x = cos\,u\ cos\,v$, $y = sen\,u\ sen\,v$

c) $z = x^2 - y^2$, $x = u - 3v$, $y = u + 2v$

d) $z = x\,e^y$, $x = uv$, $y = u - v$

10. Determine a inclinação da superfície f(x,y) no ponto P(4,2) na direção indicada. Indique também o ângulo (em graus) em relação com os respectivos eixos:

a) $f(x,y)=\sqrt{3x+2y}$, *na direção x*

b) $f(x,y)=\sqrt{3x+2y}$, *na direção y*

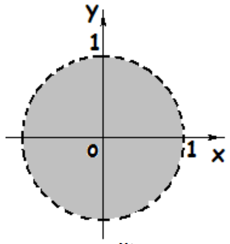
11. Determine a inclinação da superfície h(x,y) no ponto P(3,0) na direção indicada. Indique também o ângulo (em graus) em relação com os respectivos eixos:

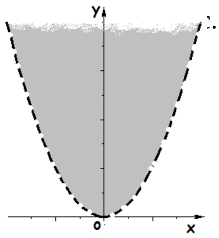
a) $h(x,y)=x\,e^{-y}+5y$, *na direção x*

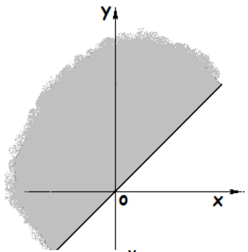
b) $h(x,y)=x\,e^{-y}+5y$, *na direção y*

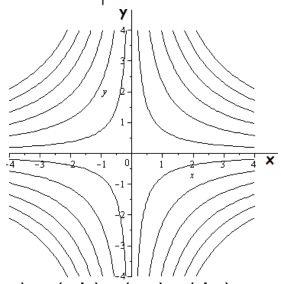
Respostas

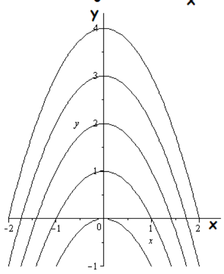
1a. $\frac{3}{8}$ 1b. $\frac{t^2}{4}-\frac{1}{t}$ 1c. 8 1d. $s^4+2s^3+s^2+4$

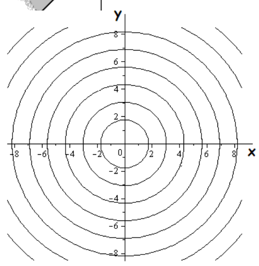
2a. 

2b. 



3a. 

3b. 



4. (c) , (d) , (a) , (b) 5. (c) , (a) , (d) , (b)

6a. $5y-2x$ 6b. $2y$ 6c. 2 e 5

7a. $9x^2y^2$, $6x^3y$, 36 , 12

7b. $2e^{2x}sen\,y$, $e^{2x}cos\,y$, 0 , 4

7c. $\frac{-\cos y}{4x\sqrt{x}}$, $-\sqrt{x}\,\cos y$, $\frac{-sen\,y}{2\sqrt{x}}$, $\frac{-sen\,y}{2\sqrt{x}}$

7d. $2xy\,e^{x^2y}$, $x^2\,e^{x^2y}$, $2y(1+2x^2y)e^{x^2y}$, $x^4\,e^{x^2y}$, $2x(1+x^2y)e^{x^2y}$

7e. $-5x^4y^4\,sen(x^5y^4)$, $-4x^5y^3\,sen(x^5y^4)$, $-20x^4y^3[x^5y^4\cos(x^5y^4)+sen(x^5y^4)]$

7f. $y\,e^{xy}\,sen\,4y^2$, $(x\,sen\,4y^2+8y\cos 4y^2)\,e^{xy}$

7g. $-\frac{y(x^2-y^2)}{(x^2+y^2)^2}$, $\frac{x(x^2-y^2)}{(x^2+y^2)^2}$

7h. $-\frac{2y}{(x-y)^2}$, $\frac{2x}{(x-y)^2}$

7i. $3e$, $2e$

7j. $-\frac{\pi}{4}$, $-\frac{1}{8}$

7k. $\frac{-16}{(4x-5y)^2}$, $\frac{-25}{(4x-5y)^2}$

7l. $15x^2y^4z^7+2y$, $21x^2y^5z^6$, $60x^2y^3z^7+2$, $105x^2y^4z^6$, $210xy^4z^6$

8a. $\frac{4t+1}{2t^2+t-1}$ 8b. $(-2\,sen\,t+cos\,t)\;cos(2\,cos\,t+sen\,t)$ 8c. $(\,5t^5+8\,)\;e^{\frac{t^5}{8}}$ 8d. $\frac{4(t^2+1)}{t(t^2+2)}$

9a. $\frac{2u^3}{\sqrt{u^4+v^2}}$, $\frac{v}{\sqrt{u^4+v^2}}$

9b. $\frac{2\;sen\,u\;cos\,u}{cos^2u+cos^2v-1}$, $\frac{2\;sen\,v\;cos\,v}{cos^2u+cos^2v-1}$

9c. $-10v$, $10(v-u)$

9d. $(1+u)\;v\;e^{u-v}$, $-(1-v)\;u\;e^{u-v}$

10a. $\frac{3}{8}$, 20.6^o

10b. $\frac{1}{4}$, 14^o

11a. 1 , 45^o

11b. 2 , 63.4^o