

Uma Equação Irracional

$\sqrt{6 + \sqrt{6 + \sqrt{6 + \dots + \sqrt{x}}}} = 3$
 $x = ?$

Iterativamente, vamos resolver esta equação partindo de t_0 ;

fazemos $t_1 = \sqrt{6 + t_0}$
 $t_2 = \sqrt{6 + t_1}$
 $\dots$
 $t_{n+1} = \sqrt{6 + t_n}$

Esta sequência converge para o valor t , pois a derivada da função $f(t) = \sqrt{6 + t}$ é

$$f'(t) = \frac{1}{2\sqrt{6+t}} < 1 \text{ para } t \geq 0.$$

Então, t é a raiz da função $t = \sqrt{6 + t} \iff t^2 = 6 + t \iff t^2 - t - 6 = 0 \iff t \in \{-2, 3\}$.

Note que, independentemente do $t_0 \geq 0$, esta sequência converge para 3.

t_n	t_{n+1}	t_n	t_{n+1}	t_n	t_{n+1}	t_n	t_{n+1}	t_n	t_{n+1}	t_n	t_{n+1}
0	2,44949	1	2,645751	3	3	5	3,316625	20	5,09902	100	10,29563
2,44949	2,906801	2,645751	2,940366	3	3	3,316625	3,052315	5,09902	3,331519	10,29563	4,036785
2,906801	2,984426	2,940366	2,990044	3	3	3,052315	3,008706	3,331519	3,054754	4,036785	3,168088
2,984426	2,997403	2,990044	2,99834	3	3	3,008706	3,001451	3,054754	3,009112	3,168088	3,027885
2,997403	2,999567	2,99834	2,999723	3	3	3,001451	3,000242	3,009112	3,001518	3,027885	3,004644
2,999567	2,999928	2,999723	2,999954	3	3	3,000242	3,00004	3,001518	3,000253	3,004644	3,000774
2,999928	2,999988	2,999954	2,999992	3	3	3,00004	3,000007	3,000253	3,000042	3,000774	3,000129
2,999988	2,999998	2,999992	2,999999	3	3	3,000007	3,000001	3,000042	3,000007	3,000129	3,000021
2,999998	3	2,999999	3	3	3	3,000001	3	3,000007	3,000001	3,000021	3,000004
3	3	3	3	3	3	3	3	3,000001	3	3,000004	3,000001
3	3	3	3	3	3	3	3	3	3	3,000001	3
3	3	3	3	3	3	3	3	3	3	3	3

Ou seja, a equação irracional proposta tem como resposta, $\forall x \in \mathbb{R}^+$