

## Equação Exponencial

$$X^{\sqrt{x}} = 2^{\sqrt{x} + 12}$$

Com  $t = \sqrt{x}$ , temos que  $t^2 = x$

$$\text{Então: } (t^2)^t = 2^{t+12} = 2^t \cdot 2^{12} = 2^t \cdot 4^6$$

$$\text{Ou: } \frac{(t^2)^t}{2^t} = 4^6 \rightarrow \left(\frac{t^2}{2}\right)^t = \frac{4^7}{2^2} = \frac{4^8}{2^4} = \left(\frac{4^2}{2}\right)^4$$

$$\text{Resumindo: } \left(\frac{t^2}{2}\right)^t = \left(\frac{4^2}{2}\right)^4 \rightarrow t = 4$$

$$\text{Agora: } x = t^2 \rightarrow x = 4^2 \rightarrow x = 16$$

# Outra maneira (numericamente):

$$(t^2)^t = 2^{t+12} \rightarrow t^{2t} = 2^{t+12}$$

$$2t \cdot \ln(t) = (t+12) \cdot \ln(2)$$

$$t_{k+1} = \frac{(t_k+12) \cdot \ln(2)}{2 \ln(t_k)}$$

Iniciando com  $t_0 > 1$ ,

$$t_k \rightarrow 4$$

t	(t+12)ln(2)	t	(t+12)ln(2)
	2.ln(t)		2.ln(t)
2	7	3,998241	4,00083
7	3,3839683	4,00083	3,999609
3,383968	4,3736362	3,999609	4,000184
4,373636	3,8456832	4,000184	3,999913
3,845683	4,0771299	3,999913	4,000041
4,07713	3,9646615	4,000041	3,999981
3,964661	4,016878	3,999981	4,000009
4,016878	3,9920942	4,000009	3,999996
3,992094	4,0037374	3,999996	4,000002
4,003737	3,9982409	4,000002	3,999999
3,998241	4,0008297	3,999999	4
4,00083	3,999609	4	4

$$\left(\frac{x}{5}\right)^x = 5^{25} \rightarrow x \cdot \ln(x/5) = 25 \cdot \ln(5)$$

Ou seja,  $x = \frac{25 \cdot \ln(5)}{\ln\left(\frac{x}{5}\right)}$

Com “chute” inicial  $x_0$  acima de 5, conseguimos uma sequência:

$$x_{n+1} = \frac{25 \cdot \ln(5)}{\ln\left(\frac{x_n}{5}\right)}$$

x	25ln(5)	x	25ln(5)
	ln(x/5)		ln(x/5)
6	220,6867	24,98655	25,00836
220,6867	10,6239	25,00836	24,99481
10,6239	53,38684	24,99481	25,00323
53,38684	16,99063	25,00323	24,998
16,99063	32,89337	24,998	25,00125
32,89337	21,35855	25,00125	24,99923
21,35855	27,71043	24,99923	25,00048
27,71043	23,49721	25,00048	24,9997
23,49721	26,00156	24,9997	25,00019
26,00156	24,40438	25,00019	24,99988
24,40438	25,38026	24,99988	25,00007
25,38026	24,76769	25,00007	24,99996
24,76769	25,14586	24,99996	25,00003
25,14586	24,90996	25,00003	24,99998
24,90996	25,05617	24,99998	25,00001
25,05617	24,96519	25,00001	24,99999
24,96519	25,02167	24,99999	25
25,02167	24,98655	25	25
24,98655	25,00836	25	25
25,00836	24,99481	25	25