

# Somas de Séries convergentes ou não

Prof. Milton Borba

Via a “soma” abaixo e resolvi escrever

$$\begin{aligned}
 S &= 1 + 2 + 4 + 8 + 16 + \dots \\
 S &= 1 + 2(1 + 2 + 4 + 8 + \dots) \\
 S &= 1 + 2S \\
 S &= -1
 \end{aligned}$$

Isto aconteceu, porque série é DIVERGENTE.

Outro caso (meu):

$$\begin{aligned}
 S &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots \\
 S &= 1 + \frac{1}{2} \left( 1 + \frac{1}{2} + \frac{1}{3} + \dots \right) + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots \\
 S &> 1 + \frac{1}{2} (S) - \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \dots \\
 S &> 1 - \frac{1}{2} + \frac{S}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \dots \\
 S &> \frac{1}{2} + \frac{S}{2} + \frac{1}{2} \left( 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \right) \\
 S &> \frac{1}{2} + \frac{S}{2} + \frac{1}{2} (S) \\
 S &> \frac{1}{2} + \frac{S}{2} + \frac{S}{2} \\
 S &> \frac{1}{2} + S
 \end{aligned}$$

$0 > \frac{1}{2}$ . Razão:  $S$  é divergente  
ver abaixo (\*)

Ainda, agora com uma série convergente:

$$\begin{aligned}
 S &= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots \\
 S &= 1 + \frac{1}{2} \left( 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots \right) \\
 S &= 1 + \frac{1}{2} (S) \Rightarrow S - \frac{S}{2} = 1 \Rightarrow \frac{S}{2} = 1 \Rightarrow S = 2
 \end{aligned}$$

(\*) Série Harmônica DIVERGE

$$\begin{aligned}
 S &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \frac{1}{13} + \frac{1}{14} + \frac{1}{15} + \frac{1}{16} + \frac{1}{17} + \dots \\
 S &> 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \dots \\
 S &> 1 + \frac{1}{2} + \frac{2}{4} + \frac{4}{8} + \frac{8}{16} + \frac{16}{32} + \dots \\
 S &> 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots \rightarrow \infty \frac{1}{2} = \infty \text{ (DIVERGE)}
 \end{aligned}$$