Exercícios de EDI – Por Séries - Prof. Milton

Resolva as seguintes Equações Diferenciais:

1) $y' - y = 0$
2) $(x+1)z' - (x+2)z = 0$
3) $f'' \pm 4f = 0$
4) $g' - g = 2t - t^2$
5) $3ty' - 2y'' + 5t^2y = \text{sen } t$
6) $u' = u/s + 1$
7) $x^2g'' + (x^2 + 5/36)g = 0$
8) $x^2y'' + xy' + x^2y = 0$
9) $(1 - t^n)\, F' = F, \quad n \in \{1, 2\}$
10) $x^2G'' + xG' + (x^2 - 1/9)\, G = 0$
11) $(1 - \theta^2)y'' - 2\theta y' + k\, y = 0, \quad k \in \{12, 20\}$
12) $ty'' + (1-t)y' - 2(1+t)y = (1-6t)e^{-t}$

Resolva os seguintes Problemas:

1) $F'' + F' = 3x^2 - x, \quad F'(0) = 0$
2) $8t^2z'' + 10t\, z' + (t - 1)z = 0, \quad z(1) = 5$
3) $f'' - 5f' + 6f = e^{2x}, \quad f(0) = 2, \quad f(1) = e^2$
4) $y'' + (t - 1)\, y' + y = t^2 - 4t, \quad y(2) = 5, \quad y'(2) = -1$
5) **Faça o gráfico da função** f, **sabendo que** $f'' + tf' + f = 2-t$; $f'(2) = -0{,}5$ **e** $f(2) = 1{,}5$.
6) **Considere que:** $f(1) = 0$, $f(3) = 10$ **e** $x.f''(x) - 4.f'(x) = f(x) + x$ **e faça o gráfico de** $f(x)$.
7) **Faça o gráfico da função** f, **tal que:** $f''(t) + 3t.f'(t) + f(t) = \cos(t)$, $f(1) = 2$ **e** $f(3{,}5) = 5$.
8) **Faça o gráfico (entre** $x = 0{,}5$ **e** $x = 2$**) da função** f, **tal que** $f''(x) + 3x.f'(x) + f(x) = e^{-x}$, $f(1)= 2$ **e** $f'(1)= -3$.
9) **Se** $f(1) = -2$, $f(4) = 3$ **e** $x.f''(x) - 4.f'(x) = f(x) + x$ **então esboce o gráfico de** $f(x)$ **no em** $[1, 3]$.
10) **Faça o gráfico da função** $y(x)$ **com valores de** x **entre** 0 **e** 2, **se** $y(0) = 0$, $y(2) = 3$ **e** $y'' - y' + xy = e^x$
11) **Represente graficamente** $f(t)$ **para** $t \in [1, 4]$ **se** $f(1) = 3$, $f(4) = 0$ **e** $t.f'' + 2.f' + t^2.f = t.\text{sen}(t)$.
12) **Se** $z(0{,}5) = 5$ **e** $z(3) = 10$, **faça o gráfico de** $z(t)$ **para** $t.z'' - 6z' + 9z = 2t$.

Exercícios de EDI – Por Séries - Prof. Milton

Respostas

1) $y = C0 + C0\,x + \frac{1}{2}\,C0\,x^2 + \frac{1}{6}\,C0\,x^3 + \frac{1}{24}\,C0\,x^4 + \frac{1}{120}\,C0\,x^5 + \frac{1}{720}\,C0\,x^6 + \frac{1}{5040}\,C0\,x^7$

2) $z = C0 + 2\,C0\,x + \frac{3}{2}\,C0\,x^2 + \frac{2}{3}\,C0\,x^3 + \frac{5}{24}\,C0\,x^4 + \frac{1}{20}\,C0\,x^5 + \frac{7}{720}\,C0\,x^6 + \frac{1}{630}\,C0\,x^7$

3) $f = C0 + C1\,x - 2\,C0\,x^2 - \frac{2}{3}\,C1\,x^3 + \frac{2}{3}\,C0\,x^4 + \frac{2}{15}\,C1\,x^5 - \frac{4}{45}\,C0\,x^6 - \frac{4}{315}\,C1\,x^7$

$f = C0 + C1\,x + 2\,C0\,x^2 + \frac{2}{3}\,C1\,x^3 + \frac{2}{3}\,C0\,x^4 + \frac{2}{15}\,C1\,x^5 + \frac{4}{45}\,C0\,x^6 + \frac{4}{315}\,C1\,x^7$

4) $g = C0 + C0\,t + \left(\frac{1}{2}\,C0 + 1\right)t^2 + \frac{1}{6}\,C0\,t^3 + \frac{1}{24}\,C0\,t^4 + \frac{1}{120}\,C0\,t^5 + \frac{1}{720}\,C0\,t^6 + \frac{1}{5040}\,C0\,t^7$

5) $y = C0 + C1\,t + \left(\frac{1}{4}\,C1 - \frac{1}{12}\right)t^3 + \frac{5}{24}\,C0\,t^4 + \left(\frac{29}{160}\,C1 - \frac{7}{480}\right)t^5 + \frac{1}{24}\,C0\,t^6$

6) $u(x) = C0 + \frac{(C0+s)\,x}{s} + \frac{\frac{1}{2}(C0+s)\,x^2}{s^2} + \frac{\frac{1}{6}(C0+s)\,x^3}{s^3} + \frac{\frac{1}{24}(C0+s)\,x^4}{s^4} + \frac{\frac{1}{120}(C0+s)\,x^5}{s^5}$

$u(s) = C0 + (C0+1)(s-1) + \frac{1}{2}(s-1)^2 - \frac{1}{6}(s-1)^3 + \frac{1}{12}(s-1)^4 - \frac{1}{20}(s-1)^5 + \frac{1}{30}(s-1)^6$

8) $y = C0 + C1\,t + \left(-\frac{1}{2}\,C1 - \frac{1}{2}\,C0\right)t^2 + \left(\frac{1}{6}\,C1 + \frac{1}{6}\,C0\right)t^3 + \left(-\frac{1}{6}\,C1 - \frac{1}{12}\,C0\right)t^4$, $t = (x\text{-}1)$

9) $F_1 = C0\,(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8)$

$F_2 = C0\,(1 + x + \frac{1}{2}x^2 + \frac{1}{2}x^3 + \frac{3}{8}x^4 + \frac{3}{8}x^5 + \frac{5}{16}x^6 + \frac{5}{16}x^7 + \frac{35}{128}x^8 + \frac{35}{128}x^9 + \frac{63}{256}x^{10})$

==========

1) $F = C0 - \frac{1}{6}x^3 + \frac{7}{24}x^4 - \frac{7}{120}x^5 + \frac{7}{720}x^6 - \frac{1}{720}x^7 + \frac{1}{5760}x^8$

2) $z = 5 + C1\,(t-1) - \frac{5}{8}\,C1\,(t-1)^2 + \left(\frac{15}{32}\,C1 - \frac{5}{48}\right)(t-1)^3 + \left(-\frac{601}{1536}\,C1 + \frac{35}{256}\right)(t-1)^4$